Dynamic Carbon Dioxide Taxation with Revenue Recycling

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Abstract

The accumulation of carbon dioxide in the atmosphere has global impacts via changing weather patterns and increasing average temperatures. These emissions, while globally important, are largely the result of decisions made at a more microeconomic level - the result of individual consumption preferences and production technologies used to make the goods we consume. Using a micro-founded DSGE framework that is calibrated with data from the United States, I model the accumulation of carbon dioxide emissions while accounting for market imperfections and frictions in the form of monopolistic competition, labor income taxation, and price-stickiness. Within this second-best setting the issues of carbon taxation and revenue recycling are addressed. I propose a dynamic, rules-based, and revenue neutral carbon tax to reduce carbon emissions and measure the resulting welfare effects. A measure of compensating variation indicates that consumers are made better off following the implementation of a dynamic carbon tax with lump-sum redistribution regardless of how the externality effect of emissions is modeled.

JEL Classification Codes: Q58, H23, Q54

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1 Introduction

Optimal environmental taxation has long been a focus of study in the economic literature, but has recently received more popular attention due to growing concerns over climate change. Although international fervor surrounding global cooperation and an intent to reduce carbon dioxide emissions is evident with the Paris Climate Accord, this accord is intentionally broad and allows for countries to pursue their own policies to reduce domestic emissions. Unfortunately, both Nordhaus (2018) and a report of the U.N. Intergovernmental Panel on Climate Change (IPCC, 2018) cast doubt on the likelihood of achieving the 2^oC goal of the Paris Agreement. And, while the breadth and

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lack of specificity of the Paris Agreement may be helpful in allowing countries to pursue their own cost-effective (or politically palatable) strategies, there is a lack of consensus among politicians and economists alike on the optimal path to take with regards to pollution taxation and revenue recycling. In his reveal of the DICE model that would eventually result in a Nobel prize, Nordhaus (1983) wrote on the importance of measuring and including a mechanism to recycle carbon tax revenues,

> "The importance of revenue recycling is surprising and striking. These findings emphasize the critical nature of designing the instruments and use of revenues in a careful manner. The tail of revenue recycling would seem to wag the dog of climate-change policy"

In order to aid policy debates as they occur globally and regionally, this paper re-addresses the problem of optimal carbon taxation and redistribution in the context of a dynamic stochastic general equilibrium (DSGE) model. DSGE modeling of global climate change has recently been cited by a number of authors as a particularly viable modeling strategy as these agent-based models confront some of the "shortcomings" of prior work (Smith, 2012). Farmer, et al. (2015) even highlight the modeling strategy as part of the "Third Wave in the Economics of Climate Change." The push toward bridging the fields of environmental economics and macroeconomics (Smith, 2012) is rather natural. Because carbon dioxide emissions (CO_2) are a transboundary pollutant, regulation of these emissions naturally seem to be a macroeconomic problem. However, CO_2 emissions are the result of individual firm and consumer actions and could thus be thought of as a 'macro-effect' that is the result of 'micro-motives'. This is one reason that DSGE modeling is an intriguing setting to analyze optimal taxation. Using a DSGE framework, this paper examines various carbon tax designs and rebatement strategies in the context of an economy in which forward-looking consumers maximize utility, firms have market power, there are labor market frictions due to taxation, there are nominal frictions in price adjustment, and the economy fluctuates along the business cycle. Both theoretically and by simulation of a micro-founded DSGE model, I show that a tax on CO_2 should be dynamic and change according to the state of the economy and the price of energy, and find that consumers are made better off (in welfare terms) under a dynamic tax relative to both a static tax or no tax at all. I further find that revenues should be returned to consumers as a lump-sum payment instead of reducing labor tax burden.

Bridging the gap between theory and policy is important, and a primary contribution of this paper is to interpret and test the derived optimal dynamic tax into a policy-rule that is derived from standard, readily available macroeconomic statistics. I do this because both consumers and firms may be averse to a tax rate that changes over time, and they may even see such a tax as capriciously changing at policymakers' whims. To that end, I develop a rules-based tax that conforms to the optimal tax that is simple to calculate and based on readily available macroeconomic aggregates - the consumer price index for energy goods and the GDP 'output gap'. The tax proposed in this paper is equivalent to an environmental 'Taylor rule' of sorts that is predictable and formulaic. The primary contribution of the present paper to the environmental economics literature is to measure the welfare impacts of this dynamic tax policy against a non-dynamic alternative in a model that includes market frictions and energy price shocks. Further, a brief discussion and analysis of the double dividend hypothesis helps to frame the full potential of a dynamic carbon tax. In a set of four policy experiments, I measure the resulting welfare effects of introducing a rules-based dynamic tax against a static tax while also toggling the method of reimbursement. Using a measure of compensating variation, I find that consumers are made better off with a dynamic carbon tax compared to no tax at all. This result holds regardless of how the externality effect of emissions is modeled (as a utility externality or a productivity externality), or how the revenues are recycled (via lump-sum transfer or by reducing labor income taxes). Moreover, This finding is robust to changes in the frictions included in the DSGE model.

1.1 Literature and background

The DSGE framework is hardly new as it is a workhorse model in the macroeconomics and monetary literature, though 'environmental macroeconomics' remains in the early stages of development (Heutel and Fischer, 2013). Calculating optimal carbon tax rates, and interactions with other pre-existing taxes, has most commonly been pursued in large-scale computable general equilibrium type models (CGE). Examples of CGE models that have been used include: DICE, GTAP, AIM, and GEM-E3. These models, however, do not consider business cycle effects. Since the preliminary works of Fischer and Springborn (2011), Heutel (2012), and Heutel and Fischer (2013), new research that considers emissions control in the context of a DSGE model has developed rapidly.

Heutel (2012) uses a real business cycle (RBC) style DSGE model to infer the relationship between persistent productivity shocks and emissions. In a RBC-type model there are no market imperfections, and markets always clear under the standard classical assumptions.[1] A key finding in Heutel (2012) is that the optimal tax is pro-cyclical, and should decrease as the economy enters into a recessionary state. Golosov, et al. (2014) use a DSGE model to develop a relatively simple formula for the optimal carbon tax. They too find that the tax should be pro-cyclical. The present paper comes to a similar conclusion. Miguel and Manzano (2006) find that the optimal tax rate on gasoline is pro-cyclical when oil consumption produces a negative externality, and that the optimal oil tax moves contrary to oil price changes in an extension to their DSGE model when they include gasoline as an input. The shared findings of prior authors that an optimal tax should move according to the state of the economy has a basis beyond theoretical work. Empirical research on carbon emissions and aggregate economic activity show that emissions are highly pro-cyclical (Doda, 2014; Khan et al. 2019). Through the use of structural vector autoregressions Khan, et al. (2019) show that there is a delayed response to income innovations and that emissions follow a hump-shaped pattern after positive economic shocks. Using a Markov switching model, Roach (2015) finds that consumer demand for gasoline, and hence emissions from gasoline, are characterized as having two latent 'states' that closely correspond with recession periods. Chan (2020) also supports the use of a dynamic tax rate on carbon dioxide, but show that the tax should complement monetary policy and not respond to fiscal policy.

DSGE models have also been used to gauge the economic costs of inaction regarding a carbon policy. Hambel, et al. (2015) makes use of a DSGE model to measure how economic growth is hampered by increasing temperatures and calculates losses due to waiting to act on a carbon policy. One of the earliest works in the environmental-macro literature is Fischer and Springborn (2009). These authors also make use of an RBC-type model, and propose emissions-intensity targets as the optimal policy against other popular mechanisms like quotas or taxes. Fischer and Springborn mention at the end of their paper that the interaction between revenue generating instruments

[1] For example, laborers are paid the value of their marginal product.

and pre-existing distortions within the economy is an avenue which merits future research. Annicchiarico and Di Dio (2015), Annicchiarico, et. al (2018) and Annicchiarico and Diluiso (2019) incorporate nominal rigidities using a 'New-Keynesian' type DSGE model. Although these models also incorporate frictions, their model differs in a number of meaningful ways from the present paper, most importantly by incorporating a monetary policy authority. These authors use this framework to discuss issues of inflation and the ability of cap-and-trade systems to provide stability to the economy and prices. Similar to the recommendation of Fischer and Springborn (2009), Annicchiarico and Di Dio (2015) discuss the need to investigate the interaction between environmental regulation and existing distortionary taxation on labor. The present work contributes to the young but growing field of environmental-macro modeling in two meaningful ways: First, by incorporating frictions in the form of imperfect competition, labor taxation, and menu-cost price rigidity; and secondly, by addressing the question of revenue recycling by allowing for the revenue generated by the carbon tax to either reduce distortionary labor taxes or be returned as a lump-sum. Importantly, Annicchiarico and Diluiso (2019), Chan (2020), and Zhang and Zhang (2020) have used the environmental DSGE modeling framework in an international setting. Chang (2020) shows how the transmission of shocks spread in a two-country setting, and further discusses the ability to cooperate to reduce emissions when there are spillover shocks. Zhang and Zhang (2020) discuss how various carbon dioxide reduction strategies impact emissions and economic activity in China.

Using revenues from an environmental tax to reduce distortionary taxes is commonly known as the double dividend. The double dividend gained widespread popularity after Oates (1995) and Goulder (1995), though it was already instantiated in the environmental and public finance literature. Oates and Goulder both set a useful platform from which many empirical and numerical studies were later based upon; Oates by noticing that a tax interaction effect could potentially counteract any double dividend effects, and Goulder by distinguishing between weak and strong dividends. The weak double dividend is a scenario in which the introduction of an environmental tax to reduce distortionary taxes leads to a larger welfare gain than when revenues are transferred in a lump-sum fashion. The strong dividend, which can only hold if the weak dividend holds, applies when welfare increases without regard to any change in environmental quality. In response to

Goulder, Bovenberg (1999) provides an updated readers guide and sheds doubt on the possibility of a double dividend existing. Other authors have joined the debate on the double dividend hypothesis, many of which cast doubt on its existence. Metcalf, et al., (2008) show that in a world with multiple distortions a weak double dividend does not necessarily hold. The results of this paper, which also incorporates multiple distortions, confirm that the weak double dividend does not hold. Under relatively strict assumptions, though, a result that is akin to the strong double dividend if found. I find that even when the stock of emissions do not affect consumer utility directly and only have a negative externality effect through changes in productivity, consumers are better off in utility-terms with a dynamic rules-based tax. I find that consumers would have to reduce their consumption by as much as 2.58% to return to the utility achieved when there is no carbon tax. This is clearly a positive signal to policy makers considering the adoption or implementation of a carbon tax. The interaction of environmental taxes and pre-existing taxes have previously been discussed dynamic general equilibrium models. Grodecka and Kuralbayeva (2014) explore the interaction of environmental taxes and labor taxes when the revenues are used to fund public capital and consumption. Barrage (2020) models carbon taxes as reducing capital income taxes and finds that the optimal carbon tax should be 20-35% lower than the implied Pigouvian tax when there are distortionary taxes. Glomm, et al. (2008) find that using a green tax to reduce capital taxation results in an increase in consumption of market goods, an efficiency dividend in their words, and an improvement in environmental quality, a green dividend in their words. Using the Glomm, et al. (2008) terminology I find that introducing a dynamic carbon tax yields both dividends.

2 Methodology

The following model seeks to represent how carbon dioxide emissions result as profit-maximizing firms and utility-maximizing consumers interact in a dynamic setting. I incorporate a number of frictions that affect this behavior: sticky pricing, market power, and labor income taxation. Prior work by Annicchiarico and Di Dio (2015) has also looked into the effects of sticky pricing, and this has been extended to both closed (Annicchiarico, Correani, Di Dio 2018) and open (Annicchiarico and Diluiso 2019) economies. The DSGE model used here is a closed economy model in which firms have market power but face adjustment costs when optimally choosing their price level. In addition, emissions (energy that pollutes upon combustion) are used as an 'input' in the production process much like the models of Copeland and Taylor (2004) in the environmental economics literature, the papers that follow in the vein of Kim and Loungani (1992) and Rotemberg and Woodford (1996) in the macroeconomics DSGE literature, and Fischer and Springborn (2011) in the environmental macroeconomics literature.

The inclusion of new frictions prove fruitful in matching the model-based impulse response functions to those seen in the empirical literature (Khan, et al., 2019). As in any model, though, a number of concessions must be made to allow for policy proposals to be discussed without the issue of confounding or compounding model intricacies. In other words, while the model includes new frictions and elements than prior authors, it does not include every possible driver of emissions or layer of complexity. For instance, the model presented here is a closed economy model. Obviously global trade and differing environmental policies will effect both agent and firm behavior, which in turn effects the global stock of carbon dioxide. This example and a number of other future model extensions that would be beneficial in the environmental macroeconomics field are discussed in the conclusion.

In what follows the baseline model and modeling assumptions are described. In the baseline model, the externality effect of emissions is borne by consumers through reduced utility, or strictly as a production externality. In other words, emissions are modeled in such a way that they either affect agents' utility function, or they affect the productive capacity of firms.[2]

2.1 Households

The representative infinitely-lived consumer wishes to maximize expected lifetime utility which is denoted by the separable utility function

$$\max E_t \sum_{t=0}^{\infty} \xi^t \Big(\frac{C_t^{1-\theta}}{1-\theta} - \chi \frac{N_t^{1-\phi}}{1-\phi} - \ln M_t \Big) \tag{1}$$

[2] There is empirical evidence that supports both ways of modeling the emissions externality. Barreca, et al., (2018) show one instance of emissions potentially affecting utility, and Heal and Park (2015) and Goodman, et al. (2019) show that emissions affect productivity.

Where ξ represents the discount rate, N_t labor supplied, M_t the stock of emissions in period t, and C_t represents consumption of the Dixit-Stiglitz aggregate over a continuum of goods indexed by j

$$C_t \equiv \Big[\int_0^1 C_{jt}^{\frac{\upsilon-1}{\upsilon}} dj \Big]^{\frac{\upsilon}{\upsilon-1}} \tag{2}$$

Note, that the production process involves new emissions. This means that consumers are made better off with more consumption (which results in more utility) at the cost of more emissions (which results in less utility). For simplicity the consumer's utility function is additively seperable between consumption, labor supply, and emissions.[3] Thus, while consumption results in more emissions there is not a marginal rate of substitution between the two 'goods'. As in any model the opportunity cost of simplification is realism. In this case realism may not be completely lost, though, as we consider how individuals may not fully consider how their marginal emissions contribute to the stock of global emissions that affects them. Indeed, Sterman and Sweeney (2007) find that graduate students at MIT have a widespread misunderstanding of the fundamental stock and flow relationships regarding carbon dioxide emissions. In any case, I discuss how results would change in light of Carbone and Smith (2008) who use a general equilibrium model to test how separability assumptions affect welfare.[4]

The stock of emissions is defined as

$$M_t = \eta M_{t-1} + m_t \tag{3}$$

where η is a parameter that represents the period-by-period decline rate of the stock of emissions, and m_t represents current period emissions that are a result of production.[5]

[3] This is a fairly safe assumption given that the pollutant studied here is transboundary and has little to no local effects beyond the effect that increased emissions have on global warming in general. Conversely the effects of nitrous oxide, for example, might be better classified as affecting consumption at the margin because there are local effects like acid rain.

[4] Carbone and Smith (2008) find that the welfare effects with a separable utility function lie within the bounds of welfare changes when environmental quality is non-separable and treated as either a complement or substitute. Measuring outcomes under various assumptions (emissions as a complement or substitute) while at the same examining market frictions and reimbursement is beyond the scope of this paper, but would merit further research

[5] Parameter values for the decay rate and the damage function are the same as in Heutel (2012) which are based on Nordhaus (2008).

The representative consumer maximizes utility subject to the budget constraint,

$$P_tC_t + K_t \leq r_tK_{t-1} + (1-\delta)K_{t-1} + (1-\tau_t)w_tN_t + T_t \tag{4}$$

which includes a net income tax τ_t[6], or transfer payment T_t, each of which is financed strictly through the revenue neutral carbon tax; wages and capital are denoted by w_t, and K_t, respectively. The first-order conditions from the households problem yield the following equilibrium conditions which govern capital supply and labor supply, respectively

$$C_t^{-\theta} = \xi E_t\big[(r_{t+1} + 1 - \delta)(\frac{1}{\pi_{t+1}})(C_{t+1}^{-\theta})\big] \tag{5}$$

$$\chi C_t^{\theta} N_t^{-\phi} = (1-\tau_t)w_t \tag{6}$$

2.2 Firms

There is a continuum of firms that produce a differentiated final-good (y_t) and sell in a monopolistically-competitive market using capital, labor, and emissions (or energy that pollutes on combustion) as inputs according to the following Cobb-Douglas production function:

$$y_t = F(K_tN_tm_t) = z_tK_t^{\alpha}N_t^{\beta}m_t^{\gamma} \tag{7}$$

where $\alpha + \beta + \gamma = 1$. Capital and labor are represented by K_t and N_t, respectively. The firm also uses energy as an input in the production process which results in emissions, m_t.[7] Fischer and Springborn (2011) also include a "polluting intermediate good" as an input in the production process. The variable, z_t, is an exogenous productivity shock that develops according to the

[6] Income tax less refunded CO_2 tax

[7] A simplification in notation has been made here. It is straightforward to convert kilowatt hours or gallons of gasoline, for example, into carbon dioxide output amounts using a carbon conversion factor. For example, using volume in gallons, V and the mega-joules per gallon of gas g we can find the amount of CO_2 emitted.

$$CO_{2,g} = V_g * (MJ/G)_g * (CO_2/MJ)_g \tag{8}$$

Thus, I simply include emissions as an input in the production process as they are a direct result of energy consumption.

autoregressive process

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{zt} \tag{9}$$

with $\epsilon_{zt} \sim N(0, \sigma_z^2)$ drawn once each period. This specification of the total factor productivity process follows the macroeconomic literature on DSGE models, and Heutel (2012).

In the production-externality version of this model, the externality effect of increased emissions is seen directly in the productivity of firms as in Heutel (2012) and emissions do not affect consumer utility. In this version of the model, the stock of emissions, M_t, reduce the productivity of the firms.

$$y_t = (1 - d(M_t)) \cdot z_t K_t^{\alpha} N_t^{\beta} m_t^{\gamma} \tag{10}$$

where d is a damage function that depends on the stock of emissions that decay according to the half-life of CO_2 emissions.[8]

The firm minimizes total costs

$$w_t N_t + r_t K_t + (p_t^m + \tau_t^m) m_t \tag{11}$$

where w_t and r_t are the real wage and the rental rate of capital, respectively. The price of energy, p_t^m, is exogenously determined and subject to shocks of the form

$$\ln p_t^m = \rho_p \ln p_{t-1}^m + \epsilon_{pt} \tag{12}$$

with $\epsilon_{pt} \sim N(0, \sigma_p^2)$. The per-unit emissions tax, τ_t^m is endogenously determined according to policy rules discussed below. Exogenous energy pricing is an assumption that is common to RBC models that include energy use in the production function, and is also used in Leduc and Sill's (2004) paper that includes price adjustment costs faced by the firm. The inclusion of energy price shocks is an innovation to the environmental-DSGE field.

Cost minimization implies the following labor, capital, and emissions demand functions

$$r_t = mc_t z_t \alpha K_t^{\alpha-1} N_t^{\beta} m_t^{\gamma} \tag{13}$$

[8] See equation 3.

$$w_t = mc_t z_t \beta K_t^{\alpha} N_t^{\beta-1} m_t^{\gamma} \tag{14}$$

$$(p_t^m + \tau_t^m) = mc_t z_t \gamma K_t^{\alpha} N_t^{\beta} m_t^{\gamma-1} \tag{15}$$

Where mc_t is the Lagrange multiplier from the cost minimization problem, and can be otherwise thought of as the real marginal cost of producing an additional unit of output.

I also include nominal price rigidity as a friction in the model. Each firm faces quadratic nominal price-adjustment costs, or menu costs, as in Rotemberg (1982).

$$\frac{\psi}{2}\Big(\frac{p_{jt}}{\pi \cdot p_{jt-1}} - 1\Big)^2 Y_t \tag{16}$$

where $\phi \geq 0$ denotes the extent of price adjustment costs. Each firm, then, seeks to maximize profits by choosing their price level

$$\max_{p_{jt}} E_t \sum_{t=0}^{\infty} \xi^t \lambda_{t+j} \Big[\frac{p_{jt} y_{jt}}{P_t} - mc_{jt} y_{jt} - \frac{\psi}{2}\Big(\frac{p_{jt}}{\pi \cdot p_{jt-1}} - 1\Big)^2 Y_t\Big] \tag{17}$$

subject to the firm's production function,

$$y_{jt} = z_t K_{jt}^{\alpha} N_{jt}^{\beta} m_{jt}^{\gamma} \tag{18}$$

and the series of demand constraints for the j^{th} firm

$$y_{jt} = \big(\frac{p_{jt}}{P_t}\big)^{-\upsilon} Y_t \tag{19}$$

Note, that as $\psi \to 0$ the problem reduces to the (nominally) frictionless problem in which prices are fully flexible and firms set a mark-up over marginal cost in accordance with the Lerner index. Finally, there is no entry or exit in the final-goods sector, and capital and labor markets are perfectly competitive.

By assumption the firm is not able to exercise market power in input markets. Thus, after equating labor supply and demand conditions (equations 6 and 14) the static "labor wedge" can be found - the difference between the firm's demand for labor based on equating the marginal product of labor with the wage rate, and the consumer's marginal rate of substitution between consumption and labor and the after-tax wage rate.[9]

$$\chi C_t^{\theta} N_t^{\phi} = (1-\tau_t) mc_t z_t \beta K_t^{\alpha} N_t^{\beta-1} m_t^{\gamma} \tag{20}$$

After equating all equilibrium conditions for the firms and households, and noting that in a symmetric equilibrium all firms will set the same price, the model is complete and an efficient pricing scheme can be derived which yields the standard new-Keynesian Phillips curve (NKPC) which is most easily seen and interpreted as a linear approximation around its steady state,

$$\widehat{\pi}_t = \frac{(\upsilon-1)}{\psi}\widehat{mc_t} + \xi E_t \widehat{\pi_{t+1}} \tag{21}$$

2.3 Ramsey Taxation

Due to the distortions in the modeled economy (e.g., the labor wedge and pricing frictions) any tax on CO_2 emissions will naturally be a second-best type policy (Lipsey and Lancaster, 1956). In order to derive optimal tax rate, and more importantly for the application here the *dynamics* of the optimal tax rate, a basic 'Ramsey planner' problem is solved.[10] The problem of the Ramsey planner is to maximize the representative consumers utility subject to the aggregate resource constraint and private-sector equilibrium conditions. The "primal" approach used here is similar to that of Chari and Kehoe (1999) who describe the method's basic framework which is to "recast the problem of choosing optimal taxes as a problem of choosing allocations subject to constraints which capture the restrictions on the type of allocations that can be supported as a competitive equilibrium for some choice of taxes" (Chari and Kehoe, 1999).

The Ramsey problem can formally be written as,

$$\max E_0 \sum_{t=0}^{\infty} \xi^t u(C_t, N_t, M_t) \tag{22}$$

[9] See Abo-Zaid (2019) for a more full discussion of the labor wedge.
[10] Emanating from Ramsey (1927)

subject to

$$\max E_0 \sum_{t=0}^{\infty} \xi^t [u_{c,t} C_t + u_{n,t} N_t + u_{m,t} M_t] = A_0 \tag{23}$$

$$C_t + K_{t+1} = z_t K_t^{\alpha} N_t^{\beta} m_t^{\gamma} + (1-\delta) K_t \tag{24}$$

Here, the left-hand-side of equation (24) is the present-value implementability constraint (PVIC) and it is equal to the constant, A_0.[11] Lagrange multipliers for the constraints are μ_1 and $\mu_{2,t}$, respectively. Note that the difference in subscripts on the two Lagrange multipliers refer to the fact that the first Lagrange multiplier is time-invariant while the other varies over time. Using the first order conditions with respect to consumption and labor it can be shown[12] that the labor income tax rate is constant and equal to

$$\tau_t = \frac{\mu_1(\theta - \phi)}{1 + \mu_1(1-\phi)} \tag{25}$$

An important feature that is of utmost importance to describing the nature of the double dividend is the way in which firms hire from input markets. If firms are able to exercise some amount of market power, then the optimal labor tax will vary over time. If firms are not able to exercise market power in input markets, then the solution to the Ramsey problem implies that the labor-income tax rate should be constant over time. This result is known as tax smoothing and can be found as early as Barro (1979). Given the modeling assumptions used here (labor markets are competitive), lump-sum transfer should be preferred over a variable income tax rate. Using Goulder's definition of strong and weak double dividends, a weak double dividend cannot be found in this model by design (tax smoothing) and a lump-sum transfer of environmental revenues will always be preferred to reducing labor taxes. Results for both transfer mechanisms are simulated in the policy experiment below, and the results of the policy experiments adhere to the received literature. Namely, that lump-sum transfer is the preferred mechanism of rebating environmental revenues.

Unlike the optimal labor tax, the tax on emissions *should* vary over time. Using the first order conditions for consumption and emissions the optimal tax on emissions is,

[11] A collection of present value utilities all of which are constant

[12] Derivation of this result, as well as the optimal carbon tax, is available in the appendix.

Table 1: Parameters

Parameter	*Explanation*	*Value*	*Source*
γ	Energy share of output	0.0818	Mean ratio of total energy expenditure to GDP
α	Capital share of output	0.33	Standard assumption
β	Labor share of output	0.5882	Calculated from former two parameters $1-\alpha-\gamma$
δ	Capital depreciation rate	0.025	Standard assumption
ψ	Rotemberg menu-cost parameter	9.9	Keen and Wang (2007)
θ	Coefficient of relative risk aversion	2	Standard assumption
ϕ	Frisch labor supply elasticity	0.33	Consistent with Frisch elasticity of labor supply of 3 Rogers and Wallenius (2009)
ξ	Quarterly discount rate	0.98267	Heutel (2012)
τ_t	Labor tax rate	0.1	Assumption
υ	Price-elasticity of demand	6	20% markup
ρ_p	Energy price autocorrelation coefficient	0.981742	Calculated
σ_p	Std. deviation of energy price innovation	0.0513	Calculated
ρ_z	Productivity autocorrelation coefficient	0.95	Heutel (2012)
σ_z	Std. deviation of TFP innovation	0.007	Heutel (2012)

$$\tau_t^M = \frac{C_t^\theta}{M_t(1+\mu_1-\theta\mu_1)} - p_t^M \tag{26}$$

From equation (27) it is apparent that the optimal emissions tax is increasing in the amount of consumption and decreasing in the price of energy.[13] This conforms with the received literature that finds that emissions taxes should be pro-cyclical, and makes intuitive sense because as the price of energy increases, emissions will decrease naturally due to the law of demand without need for further tax-induced reductions. The policy experiments of a dynamic tax-rule versus a non-dynamic alternative bleow confirm that a time-varying tax is preferred.

2.4 Parameters

Although DSGE models have scarcely been used in the environmental literature the history of DSGE modeling is rich. Thus, many of the parameters used here rely on findings and standard assumptions from the macroeconomic and monetary theory literature. There are, however, a few variables of interest that will be derived here.

Our model is calibrated with data and inputs from the United States. For example, the standard deviation of the productivity shock is calibrated in such a way that the standard deviation of output is similar to that of the standard deviation of output in the United States, a common practice in the

[13] If $\mu_1 < 1$.

DSGE literature. Moreover, the values used here are the same as Heutel (2012) and Grodecka and Kuralbayeva (2014). The persistence and standard deviation for the energy price level is calculated by estimating an ordinary least squares regression of the equation governing the price shock using a consumer price index for energy items in the United States that is produced by the Bureau of Labor Statistics. The persistence parameter, ρ_p, is 0.9817, and the standard deviation of innovations in the energy price is 0.0513.[14]

The share of output devoted to energy is also calculated. This study uses the same method and data source as Fischer and Springborn (2011), namely calculating the mean ratio of total energy expenditures to GDP from information gathered from the Energy Information Administration (EIA). The calculated value of the energy share in output is 0.0818, a slightly lower value than in Fischer and Springborn.[15]

Another variable of interest is the menu-cost parameter which has been set to a value of 9.9. This parameter corresponds with the structural estimates of Lubik and Schorfheide (2004), and Keen and Wang (2007). A value of 9.9 implies that it is optimal for roughly half of all firms to adjust prices, while others will wait to adjust due to menu-costs. Although the presence of adjustment costs causes there to be a wedge between consumption available to consumers and production, which is made greater when increasing the menu-cost parameter, the qualitative results that follow are not seriously impeded by perturbations to this parameter. The effect of this parameter on emissions is that productivity shocks cause emissions to increase for a time and then gradually decrease (a hump-shaped patter). The delayed peak in emissions is shown in the sensitivity analysis section below, and is the same type of relationship found in the empirical impulse response functions of Khan, et al. (2019). The remaining model parameters are listed in table 1.

2.5 Baseline Simulation

Before gauging how carbon taxation affects consumers, the model is simulated assuming that there is no carbon taxation. The model is solved by taking a first order approximation about the stochastic steady state as in Sims (2002). The model moments and impulse response functions are based on

[14]Similar results are found using alternative data from the Energy Information Administration.

[15]The value used here differs from Fischer and Springborn (2011) due to a longer time horizon of data

1,000 replications of 500 periods. The baseline model considered here is one in which there are no environmental taxes, but all other frictions remain. This baseline is assumed so that a compensating variation measure can be calculated to compare between the various policy experiments and the simulated economy when no such policy is enacted.

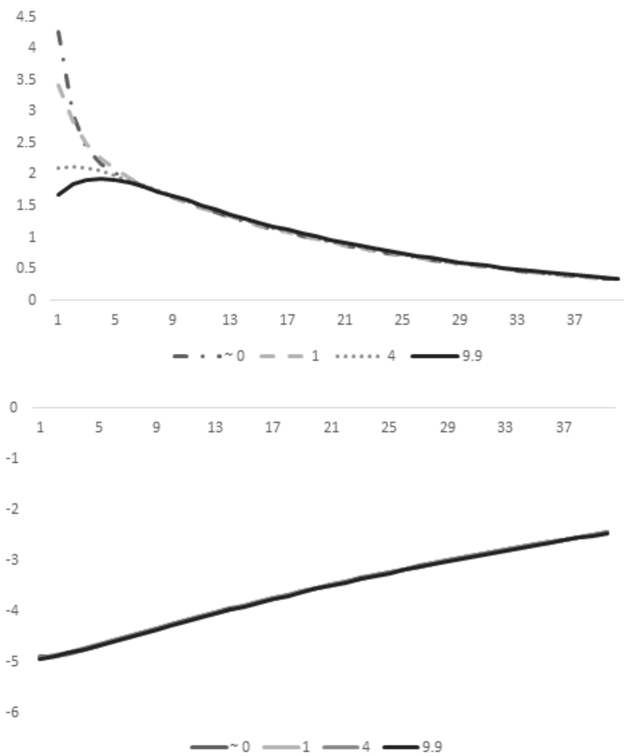
2.6 Impulse Response Functions

The response of emissions to a one standard deviation shock in TFP and a one standard deviation shock in energy price are shown below in figures 1 and 2. Each impulse response function (IRF) is normalized and shown as the percentage of the standard error of each shock. Under the parameter assumptions used here, emissions respond similarly to the empirical impulse response functions of Khan, et al. (2019). Specifically, the response of emissions to a shock in total factor productivity (TFP) very closely match the hump-shaped response found when using observed emissions data. Each impulse response function figure also shows how emissions respond to various assumptions regarding the menu-cost parameter or market power of firms holding the former constant while varying the latter, and vice versa. The model performs well under different parameter specifications with no changes to any of the qualitative results, and only minor differences in the actual numerical values that are simulated. The effect of changing each parameter on the emissions IRFs is discussed separately below, and the effect of varying each parameter on consumer welfare is discussed in the sensitivity analysis.

Many studies have shown that price rigidities play an important role in determining cost pass-through to consumers (Goldberg and Hellerstein, 2009; Nakamura and Zerom, 2010, among others). Following these authors an analysis of how the baseline model responds to perturbations of the menu-cost parameter is of particular interest. Figure 1 shows the sensitivity of the emissions IRFs to changes in the menu cost parameter, holding all other parameters constant. The menu-cost parameter, ψ, ranges from a very flexible price environment ($\psi \approx 0$) to a less flexible environment ($\psi = 9.9$). Note, that the baseline model assumes a menu-cost value of 9.9. This corresponds to a Calvo pricing model in which half of all firms readjust their prices instantaneously (Keen and Wang, 2007).

For each of the impulse response functions we can see that varying the menu-cost parameter has

Figure 1. Emissions Under Differing Menu-cost Assumptions



Notes: TFP Shock (top) Energy Price Shock (bottom); Vertical Axis measures the percentage of the standard error of each shock.

a moderate effect on the values of the IRF and changes the shape of the IRF in a manner that is expected. Following a TFP shock when menu-costs are quite low ($\psi \approx 0$) emissions respond quickly and gradually decrease, whereas in a less flexible price environment ($\psi = 9.9$) emissions have the hump-shaped response that matches the response found in the empirical literature.[16] Following an energy price shock it is evident that emissions will decrease on impact, but gradually return to pre-shock levels. Menu-costs variations do not affect the shape or structure of the response to an energy price shock.

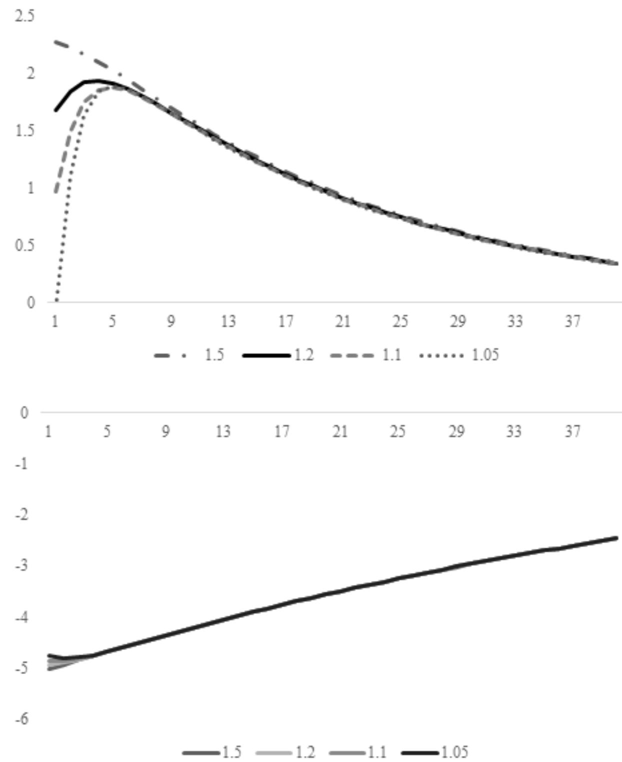
The second set of impulse response functions (Figure 2) show how emissions respond to the exogenous TFP and energy price shocks while varying the amount of market power firms have (holding all else constant). In Figure 2, market power varies from firms using a large steady-state mark-up factor of 1.5 to a small mark-up of 1.05.[17]

In figure 2 it is clear that emissions have a delayed response to TFP shocks when market power is lower. An explanation as to why a delay occurs can be found in the coordination between price adjustment costs and market power. Firms with a small amount of market power cannot adjust prices as easily, and are not as insulated from exogenous shocks by holding price well above marginal cost. Thus, they are not able to pass along price differences by exerting market power and may produce sub-optimally for some time. This in turn leads to a delayed response in utilizing more emissions to produce larger quantities (in the case of a positive TFP shock). After four periods have transpired, the IRF for each market-power assumption begins to converge to the same value. This makes intuitive sense given the assumption that 50% of firms will choose to re-adjust their prices each period; four periods after a shock roughly 94% of firms will have re-optimized their price. Market power has very little effect on changing the response of emissions to an energy price shock. There is a small hump-shaped pattern when mark-ups are low, but not as drastic of a difference when there is a TFP shock.

[16] Note that ($\psi > 20$) is not pictured because the response is not feasible. This is due to the fact that the model does not converge for values larger than 20. This is not surprising, though, as one would not expect menu costs to delay price adjustment for more than a year.

[17] The formula for calculating the steady-state markup is $\frac{\upsilon}{\upsilon - 1}$

Figure 2. Emissions Under Differing Market Power Assumptions



Notes: TFP Shock (top) Energy Price Shock (bottom); Vertical Axis measures the percentage of the standard error of each shock.

3 Carbon Taxation Policy Experiments - Simulation Results

The work of prior authors in the environmental-DSGE literature indicate that CO_2 taxes should be flexible in nature and oscillate according to the business cycle. A contribution of the present paper to this literature is to measure the welfare impacts of a dynamic tax policy against a non-dynamic alternative in a model that includes market frictions and energy price shocks. Further, a brief discussion and analysis of the double dividend hypothesis helps to frame the full potential of a dynamic carbon tax. In this section, four revenue-neutral carbon tax policy 'experiments' are run:

- time-invariant emissions tax is returned by reducing marginal labor taxes
- dynamic emissions tax is returned by reducing marginal labor taxes
- time-invariant emissions tax is returned via lump-sum transfer
- dynamic emissions tax is returned via lump-sum transfer

Importantly, these policy scenarios allow for pairwise comparisons to be made both between how revenues are returned (holding constant the tax policy) and the dynamic versus the constant tax (holding constant the method of revenue reimbursement).

In order to discuss whether a dynamic tax is preferred to a flat tax, and how environmental revenues should be returned to consumers, I model a carbon tax that is based on the goals set forth by the United States at the 2009 climate talk in Copenhagen, Denmark. The United States set a goal to reach 17% reductions in CO_2 emissions from 2005 levels by the year 2020.[18] In order to reach this level of CO_2 reductions in the steady-state, compared to the baseline model with no environmental taxation, a tax of 23% is required.

As the Ramsey planner's problem and prior authors have shown, the optimal environmental tax should increase as the economy expands and decrease in response to energy price shocks. That is to say, in the presence of positive productivity shocks the optimal tax should increase to dissuade the heightened use of energy, and should decrease following spikes in energy prices because energy demand will naturally be lower due to price effects.[19] Following these results I propose a dynamic

[18] The recent Paris climate agreement is centered on keeping the global temperature rise below 2 degrees Celsius; not a specific CO_2 reduction. Moreover, the United States has withdrawn from this accord.

[19] In general, as prices increase, quantity demanded naturally falls.

tax rule that would be predictable and forecastable for firms and policy-makers to mitigate future policy uncertainties. The tax rule is designed to be an environmental 'Taylor-rule' of sorts that depends on deviations from the steady-state level of output and the steady-state energy price. The reason for a rules-based dynamic tax is to stave off concerns that a tax increase or decrease is spurious or at the whim of a policy-maker. Specifically, the dynamic tax proposed here increases as output increases, and decreases as energy price increases. This rule is developed to offer policy-makers a viable policy-rule that is determined by readily available macroeconomic aggregates. I compare the welfare results of the dynamic tax to that of a no policy environment as well as the welfare that would occur if a static carbon tax were implemented. The tax rule is expressed below in equation (28),

$$\tau_t = \omega_t + \beta_y \ln(y_t - \overline{y}) - \beta_p \ln(p_t^m - \overline{p}^m) \tag{27}$$

where ω_t denotes the value of the non-dynamic tax so that the two taxes are exactly the same if the economy is in the steady state, bars above output and the price of energy indicate steady-state levels, and the variables, β_y and β_p are time-invariant calculated weights that maximize average utility of the consumer.[20]

To make a comparisons of each policy alternative, as well as to compare against the baseline of no taxation, it is natural to compare the various amounts of utility that each policy scenario results in. Under all policy scenarios I find that utility increases following a dynamic CO_2 tax. Utility is an ordinal measure, though, so the degree to which a CO_2 tax-policy is preferred over having no policy at all remains unclear. To compare policies I use a measure of compensating variation (CV) as in Gali (2008). CV is calculated by finding the amount that consumption would need to change to return the representative consumer to the amount of utility achieved in the no-policy-scenario holding the level of emissions and labor-supplied from the policy scenario constant. In other words, the CV measure calculates whether a consumer would need to increase or reduce consumption to be as well off, in utility terms, as she was in a world without environmental taxation. The calculation

[20] $\beta_y = \beta_p = 0.6$. Solved for by iterative maximization of the utility function. Interestingly, utility is only maximized when these weights are equal to each other. This is driven by the assumption that although a shock to either energy or TFP may have a larger standard deviation or last longer, the frequency or probability of a shock is the same for both exogenous variables.

is expressed below in equation (29), and is listed in the table in percentage change terms.

$$CV \equiv \%\Delta C^{Tax} \tag{28}$$

such that

$$U(C^{NP}, N^{NP}, M^{NP}) = U(C^{Tax}, N^{Tax}, M^{Tax}) \tag{29}$$

Superscript, NP denotes the no policy (baseline) scenario, and superscript, Tax indicates the various tax and revenue rebating scenarios. Each value is the average over 500 periods in 1,000 replications.

3.1 Policy Experiment Results

Regardless of whether emissions affect utility or productivity, I find that the preferred taxation and rebate strategy is indeed a dynamic emissions tax returned as a lump-sum payment. Moreover, I find that a dynamic emissions tax policy is welfare enhancing, compared to no policy at all, regardless of how the emissions externality is modeled.

When emissions are a utility externality, in each of the four tax and rebate scenarios I find that consumers must forego consumption to return to the utility realized when there is no tax policy. This indicates that consumers prefer having a carbon tax compared to the baseline of no carbon tax. Using the CV calculation and comparing across policy-scenarios it is easy to see two major results: (i) lump-sum distribution is the preferred mechanism to rebate environmental revenues — consumers would need to reduce consumption by about 2% more than when the labor tax is reduced — and (ii) the dynamic tax is preferred over the non-dynamic alternative regardless of the method of reimbursement. Given the findings from the Ramsey-planner problem it is not surprising that returning environmental revenues from a dynamic tax in a lump-sum fashion is strictly preferred over all other possibilities. Under this method of taxation and reimbursement consumers would have to decrease consumption by 9.08% to maintain pre-policy utility. The improvement in consumer well-being is robust to changes in modeling assumptions including varying the menu-cost and market power assumptions.

Table 2. Simulation Results

	Decrease Labor Tax		Lump-Sum Transfer	
	Fixed	*Dynamic*	*Fixed*	*Dynamic*
Utility Externality				
Compensating Var.	-6.66	-7.86	-7.69	-9.08
Change in Emissions	-16.61	-21.18	-16.44	-20.89
Production Externality				
Compensating Var.	0.83	-1.25	0.73	-2.58
Change in Emissions	-16.52	-20.33	-16.37	-20.10

Table 2: Notes: Each measure is relative to the baseline simulation which includes all frictions and no taxation, and is the average of 500 periods simulated 1,000 times. Compensating variation and change in emissions expressed in percentage change terms. A negative CV value implies an increase in consumer welfare.

A note is in order to further contextualize these welfare findings. Carbone and Smith (2008) investigate how consumer welfare is affected in CGE models when environmental quality and consumption are not additively separable as I have modeled them here. The findings of Carbone and Smith (2008) imply that CV would greater if environmental quality is non-separable and treated as a complement, and less if treated as a substitute. Thus, I consider the 9.8% CV measure as an intermediate value because it could vary in either direction depending on a consumer's preferences.

Prior authors in the environmental-DSGE field have modeled the externality effect of carbon dioxide emissions as a productivity externality, and there is a growing literature of empirical evidence to back this assumption (Dell, et al., 2009; Zivin, et al., 2017; Bishop, et al., 2018; Goodman, et al., 2018). The lower panel of table 2 presents CV measurements when productivity, *and not utility*, are impacted by the stock of emissions. The way that emissions can impact CV in this model is that, all else equal, total production is lower when there are more emissions (which are a result of prior period consumption). Thus, equilibrium consumption and prices are changed contemporaneously and in the future according emissions rates and the half-life of CO_2.

In the productivity externality version of the model I find that consumers welfare is better off following a dynamic tax, and worse-off following the implementation of a static emissions tax. In other words, for the flat emissions tax scenario consumers would have to be compensated with extra consumption to return them to their pre-policy level of utility. The amount these consumers would have to be compensated in extra consumption is less than 1% for each refunding scenario. This difference in welfare results is important for policy-makers. If consumer utility is not directly affected by global climatic change, then a static tax may result in an, albeit small, welfare loss. However, the policy experiment results do show that consumers are made better off following the implementation of the dynamic rules-based tax. I find that consumers would need to forego consumption by up to 2.58 % to return to the no-policy level of utility.

Goulder (1995) provides a useful starting point for any discussion on the double dividend hypothesis. There are many possible reasons why the weak double dividend hypothesis does not hold for the modeled economy here. First, as Deaton (1979), Babiker, Metcalf, and Reilly (2003), and Metcalf, et al. (2008) show, a lump-sum transfer may be preferred because any efforts to reduce distortionary taxes causes a larger gap between the after tax prices of energy and labor. Second, a common finding in the macroeconomics literature on tax-smoothing suggests that when the degree of market power is unchanging, the optimal labor tax rate is constant over time. Thus, any effort to reduce distortionary taxes is suboptimal in the first place and the planner should instead solve for an optimal labor tax with the knowledge that environmental revenues will be refunded in a lump-sum fashion. Glomm, et al. (2008) distinguishes their double dividend in terms of an efficiency dividend, an increase in consumption, and a green dividend, an increase in environmental quality. Using the Glomm, et al. (2008) definition of the double dividend, I find that a dynamic emissions the tax does yield a double dividend, albeit by an incremental margin. This is a major benefit to the policy-maker because as Hsu (2011) notes, the major impediment to a carbon tax is simply its political feasibility. Given these results, introducing a tax on emissions is a policy that can be justified as being beneficial to consumers.

3.2 Sensitivity of Results

This section addresses how consumer welfare, as measured by compensating variation, is affected by modeling assumptions introduced here. Specifically, I test the sensitivity of compensating variation to changes in both the menu-cost and market power parameters. This robustness exercise is important because as Annicchiarico and Di Dio (2015) show, welfare tends to be slightly higher

Table 3: Sensitivity Analysis

% Adjusting	Parameters	Decrease Labor Tax		Lump-Sum Transfer	
		Fixed	*Dynamic*	*Fixed*	*Dynamic*
50	$\upsilon = 4; \psi = 6.6$	-6.7	-8.04	-7.43	-9.12
75	$\upsilon = 4; \psi = 1.5$	-7.23	-8.57	-9.1	-10.53
50	$\upsilon = 6; \psi = 9.9$	-6.66	-7.86	-7.69	-9.08
75	$\upsilon = 6; \psi = 2.2$	-7.14	-8.47	-9.04	-10.47
50	$\upsilon = 11; \psi = 19.8$	-5.95	-7.05	-6.7	-8.45
75	$\upsilon = 11; \psi = 4.4$	-6.96	-8.27	-8.96	-10.35

Table 4: CV measure shown in percentage change; Sensitivity is comparable across different menu-cost assumptions holding market power constant (grouped rows), or different market power assumptions holding menu-cost constant (every other row)

with a tax if the degree of price stickiness is not too high, and further find that the optimal tax response to shocks is significantly affected by the degree to which prices are able to adjust.

Table 3 displays how compensating variation (CV) responds to differences in menu-costs or market power for the utility-externality model. I use the results of Keen and Wang (2007) to establish changes in the menu-cost parameter as differences in the percent of firms that will choose to re-optimize prices each period.[21] This allows me to compare across market power assumptions holding menu-costs constant (and vice versa).

Each grouping of rows shows how CV changes as menu-costs decrease - holding market power (υ) constant. I find that decreasing menu-costs results in a higher CV regardless of the market power assumption. This finding makes intuitive sense because we would expect consumption to be higher when firms are better able to optimize their prices to market conditions. For example, it is less likely that prices are inefficiently high when prices are more flexible. This makes sense in relation to the findings of Annicchiarico and Di Dio (2015) who find that the optimal policy response of taxation is stronger when prices are stickier.

Comparing every other row of the table allows for a comparison between market power assumptions, holding the amount of re-optimizing firms constant. I find that differences in market power do not affect the results as much as differences in menu-costs. In fact, the CV for a dynamic tax that returns revenues in a lump-sum is largely unchanged across market power assumptions.

[21] This is similar in interpretation to nominal frictions that are modeled according to a Calvo pricing model where a random percent of firms update their pricing each period.

Interestingly, I do find that the change in CV due to firms being able to adjust their prices is made more drastic as market power decreases. This is an important finding because it shows that the coordination of market power rigidities and nominal frictions affect the overall welfare of consumers in this model. The baseline model assumes a steady-state markup of 20% and that only 50% of firms are able to re-optimize their prices each period. The simulated tax policy would result in larger welfare gains if a less conservative assumption on menu-costs was made.

4 Conclusions

This paper has furthered a growing field of research that examines the problem of carbon dioxide mitigation in a dynamic and stochastic environment by incorporating several market frictions. In addition to exploring methodological differences, this paper reinforces the finding of prior authors that carbon taxes should be dynamic, and further finds that rebating tax revenues to consumers in the form of a lump-sum subsidy is preferred to reducing labor taxes. This is shown using four simulated policy-experiments in which a rules-based dynamic tax is implemented. I find that consumers would have to forego consumption by 2.6-9.1% to remain as well off as they were in a world without a carbon tax when there is a rules-based dynamic tax with revenues redistributed as a lump-sum. In other words, consumers are made better off following the implementation of new carbon dioxide taxation. The positive result that consumer welfare is improved following the implementation of a carbon tax is robust to changes in both the modeling of the externality effect of emissions and the modeling of model frictions. This is a boon to policy-makers that wish to address the pressing problem of global climate change because it indicates that the implementation of an environmental tax can be beneficial in utility terms over a wide range of assumptions and market conditions.

While a theoretical analysis of the problem of carbon taxation is certainly necessary for the ongoing discussion and research on confronting climate change, a shortcoming of this paper lies in the fact that a representative consumer is used to express the interests of a population that is actually quite varied. For instance, the distributional effects of returning revenues in a lump-sum are of interest because lower income individuals typically have a much higher propensity to consume than their more wealthy counterparts. Thus, returning revenues to these individuals may spur economic activity by more than this paper is able to analyze. Similarly, in the modeled economy presented here consumers are forced to consume an aggregate good that represents a basket of goods. If revenues were instead used to incentivize cleaner consumption from an emissions standpoint, then a larger 'green dividend' may actually be realized; a triple dividend of sorts. Evidence of this type of policy having success can readily be seen by the many accomplishments of states involved in the Regional Greenhouse Gas Initiative of the northeast United States. Many states in this coalition use revenues from carbon permit auctions to further environmental projects like funding energy audits for the poor, installing solar panels on universities, and replacing government fleet vehicles with lower emission vehicles (RGGI, 2013). Further research that considers this "third" dividend in a dynamic and stochastic environment would certainly be of merit.

5 Appendix: Ramsey Tax Derivation

The Ramsey problem from section 3.3 is

$$\max E_0 \sum_{t=0}^{\infty} \xi^t u(C_t, N_t, M_t) \tag{1}$$

subject to

$$\max E_0 \sum_{t=0}^{\infty} \xi^t [u_{c,t} C_t + u_{n,t} N_t + u_{m,t} M_t] = A_0 \tag{2}$$

$$C_t + K_{t+1} = z_t K_t^{\alpha} N_t^{\beta} m_t^{\gamma} + (1-\delta) K_t \tag{3}$$

Where the constraints on the present-value implement-ability constraint (2) and the budget constraint (3) are μ_1 and μ_2 respectively. Note, that the former constraint is time-invariant while the latter constraint may vary over time; for ease of exposition, the time subscript on μ_2 has been eliminated. Below, derivatives of the utility function, U, and the production function, F, are with respect to the letter subscript (e.g. U_C is the first partial derivative of utility with respect to consumption).

5.1 Optimal Labor Tax

Taking first-order conditions with respect to consumption yields

$$U_C(1+\mu_1) + \mu_1 U_{CC} C_t = \mu_2 \tag{4}$$

Using the functional form from before (CRRA) this can be written as

$$C_t^{-\theta}(1+\mu_1) + \mu_1(-\theta C_t^{-\theta}) = \mu_2 \tag{5}$$

Which can be transformed to

$$C_t^{-\theta}(1+\mu_1 - \theta\mu_1) = \mu_2 \tag{6}$$

or more generally,

$$U_C(1 + \mu_1 - \theta\mu_1) = \mu_2 \tag{7}$$

Moving to the first order condition on labor,

$$U_N + \mu_2 F_N + \mu_1[U_{NN}N + U_N] = 0 \tag{8}$$

Using the functional form from before this can be written as

$$-\chi N_t^{-\phi} + \mu_1(-\chi N_t^{-\phi})(1 - \phi) = -\mu_2(F_N) \tag{9}$$

or more generally,

$$U_N[1 + \mu_1(1 - \phi)] = -\mu_2(F_N) \tag{10}$$

Substituting equation (7) into (10) yields,

$$\frac{-U_N}{U_C} = (F_N)\frac{(1 + \mu_1 - \theta\mu_1)}{[1 + \mu_1(1 - \phi)]} \tag{11}$$

Utilizing the labor market equilibrium condition from section 3,

$$\frac{-U_N}{U_C} = (F_N)(1 - \tau_t) \tag{12}$$

Equations (11) and (12) can be combined to yield,

$$\frac{(1 + \mu_1 - \theta\mu_1)}{[1 + \mu_1(1 - \phi)]} = (1 - \tau_t) \tag{13}$$

After some basic manipulation, this can be transformed to express the optimal tax rate as a function of the PVIC constraint which is a comprised of time-invariant terms: μ_1, ϕ, and θ.

$$\tau_t = \frac{\mu_1(\theta - \phi)}{1 + \mu_1(1 - \phi)} \tag{14}$$

5.2 Optimal Emissions Tax

The first-order condition with respect to emissions is,

$$U_M + \mu_1(U_{MM}M + U_M) + \mu_2(F_M) = 0 \tag{15}$$

Using the logarithmic specification from before, the second term in (15) can be eliminated. Also, substituting the equilibrium condition for the derivative of the production function with respect to emissions yields,

$$U_M + \mu_2(p^m + \tau_t^m) = 0 \tag{16}$$

Substituting (7) into (15) yields,

$$U_M + U_C(1 + \mu_1 - \theta\mu_1)(p_t^m + \tau_t^m) = 0 \tag{17}$$

Utilizing the functional forms from before this can be re-written as

$$C_t^{-\theta}(1 + \mu_1 - \theta\mu_1)(p_t^m + \tau_t^m) = \frac{1}{M_t} \tag{18}$$

After some basic manipulation the optimal environmental tax can be expressed in terms of the price of energy, consumption, emissions, and the constraint on the PVIC.

$$\tau_t^m = \frac{C_t^\theta}{M_t(1 + \mu_1 - \theta\mu_1)} - p_t^m \tag{19}$$

From (19) it is clear that the optimal environmental tax varies over time and is increasing in consumption, decreasing in the price of energy, and increasing in the amount of emissions if $\mu_1 > 1$.

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