Homework 3

ECON 5453

September 28, 2020

1.

The Department of Economics at Metrics University randomly sampled 22 MSBA students at the beginning of the Fall 2018 semester and obtained data on the following variables: the current grade point average of student "i" (call this variable "GPA_i"); the grade point average of the student upon graduating from their undergraduate (call this variable "$UGPA_i$"); the average number of hours per week that student i spent at the bar (call this variable "BAR_i"); and the average number of hours per week that student i studied (call this variable "$STUDY_i$"). The Department of Economics at Metrics University estimated (by OLS) three alternative regression models using these variables, and their results are shown below:

$$(1.)\ GPA_i = 0.90 + 0.5UGPA_i + \hat{u}_i \quad \text{S.E. of } \hat{\beta}_0 = 0.20 \quad \text{S.E. of } \hat{\beta}_1 = 0.20$$

$$(2.)\ GPA_i = 2.0 - 0.1BAR_i + \hat{u}_i \quad \text{S.E. of } \hat{\beta}_0 = 0.14 \quad \text{S.E. of } \hat{\beta}_1 = 0.05$$

$$(3.)\ GPA_i = 1.6 + 0.3STUDY_i + \hat{u}_i \quad \text{S.E. of } \hat{\beta}_0 = 0.90 \quad \text{S.E. of } \hat{\beta}_1 = 0.10$$

(a) For all three equations above, compute the t-test statistics for the following null and alternative hypotheses: $H_0 : \beta_0 = 1$ *versus* $H_A : \beta_0 \neq 1$. Evaluate the test at the 5% significance level.

For each test, the t statistic is formed as $t = \dfrac{\hat{\beta}_0 - 1}{S.E.of\hat{\beta}_0}$ *which equates to -0.5; 7.14; and 0.67 for equations 1-3, respectively. The degrees of freedom are* $(N-[K+1]) = 22-2 = 20$. *To evaluate the test, note that we have a two-tailed test, Thus, the critical values at the 5% significance level are -2.086 and +2.086. Thus, for equations (1.) and (3.) we do not reject the null; for equation (2.) we do reject the null.*

(b) For equations (1.) and (3.) above, compute the t-test statistics for the following null and alternative hypotheses: $H_0 : \beta_1 \leq 0$ *versus* $H_A : \beta_1 > 0$. Evaluate the test at the 5% significance level.

For each test, the t statistic is formed as $t = \dfrac{\hat{\beta}_1 - 0}{S.E.of\hat{\beta}_1}$ *which equates to 2.5 for equation (1.) and 3 for equation (3.). The degrees of freedom are* $(N - [K + 1]) = 22 - 2 = 20$. *To evaluate the test, note that we have a one-tailed (right) test, Thus, the critical values at the 5% significance level is 1.725. In both cases we reject the null*

(c) For equation (2.) above, compute the t-test statistic for the following null and alternative hypotheses: $H_0 : \beta_1 \geq 0$ *versus* $H_A : \beta_1 < 0$. Evaluate the test at the 10% significance level.

For the test, the t statistic is formed as $t = \dfrac{\hat{\beta}_1 - 0}{S.E.of\hat{\beta}_1}$ *which equates to -2 for equation (2.). The degrees of freedom are* $(N - [K+1]) = 22 - 2 = 20$. *To evaluate the test, note that we have a one-tailed (left) test, Thus, the critical values at the 10% significance level is -1.325. Thus, we reject the null*

2.

Suppose you estimate the following model for housing prices in the San Francisco Bay area:

$$\hat{HouseP_i} = \underset{(0.1.2)}{5.2} \quad \underset{(0.24)}{-0.725PropTax_i} \quad \underset{(0.25)}{+0.547SQFT_i} \quad \underset{(0.0005)}{+0.00073Inc_i} \quad \underset{(0.02)}{+0.064Age_i} \quad \underset{(0.002)}{-0.0043Travel_i}$$

Where $HouseP_i$ represents the price of house "i" measured in thousands of dollars: "PropTax" represents the amount of property taxes for house i in dollars; "SQFT" represents the square footage of house i; "Inc" represents mean income (in dollars) of families in the neighborhood; "Age" represents the number of years since home i was built; and "Travel" represents the distance the home is from downtown San Francisco. The numbers in parentheses beneath the parameter estimates are the corresponding (estimated) standard errors. Note, N = 15. Use the above information to evaluate each of the tests described below. In each case, be sure to: write out the null and alternative hypotheses in terms of betas and constants; compute the numerical value of the test; note the numerical values of the degrees of freedom and critical value(s) of the test; and, indicate whether or not the null hypothesis would be rejected.

(a) Test whether or not the variable "PropTax" has a statistically significant effect on the price of a house. Use the 5% level of significance to evaluate the test.

$t = -3.02$, $DF = 9$, *Critical value for two-tailed test at 5% significance level is* ± 2.262. *Therefore, we reject* H_0.

(b) Test whether or not the variable "SQFT" has a statistically significant and POSITIVE effect on the price of a house. Use the 5% level of significance to evaluate the test.

$t = 2.188$, $DF = 9$, *Critical value for one-tailed test at 5% significance level is* 1.833. *Therefore, we reject* H_0.

(c) Test whether or not the variable "Travel" has a statistically significant and NEGATIVE effect on the price of a house. Use the 10% level of significance to evaluate the test.

$t = -2.15$, $DF = 9$, *Critical value for one-tailed test at 10% significance level is* 1.383. *Therefore, we reject* H_0.

(d) Test whether or not the variable "Age" has a statistically significant one-to-one effect on the price of a house. Use the 1% level of significance to evaluate the test.

$t = -46.81$, $DF = 9$, *Critical value for two-tailed test at 1% significance level is* ± 2.764. *Therefore, we strongly reject* H_0.