

# Homework 1 - ECON 5453

## Solutions

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September 1, 2019

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### 1.

Consider the parameter  $\beta$  which is normally distributed from a population which has variance equal to  $\sigma^2$ . Next, consider two estimators,  $\hat{\beta}$  and  $\tilde{\beta}$ , of this parameter  $\beta$ . The two estimators have the following characteristics:

$$\hat{\beta} \sim N(\beta, \sigma^2) \text{ and } \tilde{\beta} \sim N(2\beta, 4\sigma^2)$$

- (a) According to this notation, what is the expected value of  $\hat{\beta}$ ? According to this notation, what is the expected value of  $\tilde{\beta}$ ?

*The expected value of  $\hat{\beta}$  is  $\beta$ ; the expected value of  $\tilde{\beta}$  is  $2\beta$*

- (b) Is  $\hat{\beta}$  an unbiased estimator of  $\beta$ ? Explain and give proper evidence to support your answer (convince me that you are not just guessing).

*Since  $E[\hat{\beta}]$  is in fact equal to the true population parameter,  $\beta$ , this estimator is unbiased.*

- (c) Is  $\tilde{\beta}$  an unbiased estimator of  $\beta$ ? Explain and give proper evidence to support your answer (convince me that you are not just guessing).

*Since  $E[\tilde{\beta}]$  is NOT equal to the true population parameter (it is equal to  $2\beta$ ) this estimator is NOT unbiased.*

- (d) According to this notation, what is the variance of  $\hat{\beta}$ ? According to this notation, what is the variance of  $\tilde{\beta}$ ?

*The variance of  $\hat{\beta}$  is  $\sigma^2$ ; the variance of  $\tilde{\beta}$  is  $4\sigma^2$ .*

- (e) Which of these two estimators is relatively more efficient? Explain.

*Since  $\text{var}(\hat{\beta}) < \text{var}(\tilde{\beta})$ , it follows that  $\hat{\beta}$  is relatively more efficient.*

### 2.

You are the owner of a production plant that produces iPhones. In a typical week, the number of iPhones produced is a normally distributed random variable with an average production level of 174 and a standard deviation of 9.

- (a) What is the probability that the plant will produce between 160 and 180 iPhones in any given week?

*68.92%*

- (b) Due to increased advertising in recent weeks, you expect the demand for iPhones next week to exceed 190. What is the probability that the plant will produce 190 iPhones or more?

*3.75%*

- (c) The manager of your iPhone plant wants to tell you with a 95% probability that production levels will fall within some range, say from  $X_{lower}$  to  $X_{upper}$ . What should these values be?

$X_L = 156.36$  and  $X_U = 191.64$

### 3.

You have been hired by the East Coast Tourists' Association to analyze the incidence of shark attacks on humans at US beaches. Your research shows that, in any given year, shark attacks is a normally distributed variable with an average of 45 attacks per year and a variance of 64. Use this information to answer each of the following questions.

- (a) What is the probability that, in any given year, there will be less than 27 shark attacks?

*1.22%*

- (b) What is the probability that, in any given year, there will be between 33 and 55 shark attacks?

*$(0.8944 - 0.0668) = 0.8276$  or  $82.76\%$*

- (c) The President of the East Coast Tourists' Association wants to know how many shark attacks to expect in any given year within a range from  $X_{lower}$  to  $X_{upper}$ , with a 78.5% probability. That is, she wants to say with a 78.5% probability that there will be between  $X_{lower}$  and  $X_{upper}$  shark attacks. What should the values of  $X_{lower}$  and  $X_{upper}$  be?

$X_L = 35.64$  and  $X_U = 54.36$