Hidden Regimes and the Demand for Carbon Dioxide from Motor-Gasoline

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July 15, 2014

Abstract

Perhaps the most familiar source of anthropogenic carbon dioxide (CO\textsubscript{2}) emissions stem from vehicle-use. For this reason it is likely that consumer demand for CO\textsubscript{2} emissions from vehicle-use are quite responsive to the overall state of the economy. Using a structurally identified Markov-switching demand model I find that CO\textsubscript{2} emissions respond asymmetrically to changes in income and the price of gasoline in expansionary and contracting states of the economy. As the Lucas Critique warns, we cannot expect emissions demand to respond to policy measures in the same manner across differing regimes. The findings of this paper indicate that flexible policy instruments have the potential to mitigate undue burden on consumers and producers compared to their static counterparts.

\textbf{Keywords:} Carbon Dioxide, Markov Switching, Motor-Gasoline

\textbf{JEL Classifications:} C510, Q410, Q430, Q530

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1. Introduction

The scope and method of energy use in the United States has changed vastly in the last four decades to accommodate technological and societal progress. Indeed, since the oil embargo of 1973 the manner in which consumers demand gasoline in the United States has evolved greatly. Consumers, firms, and policy-makers alike have confronted issues of energy scarcity and abundance, the introduction of new plant-based fuels, and confronted growing concern for the environmental impact of emissions that are a natural byproduct of the combustion process. Recent concerns over anthropogenic climate change have brought the issues of energy production and consumption to the forefront of debates at both the national and international level. For instance, the Intergovernmental Panel on Climate Change (IPCC) recently established that it “is extremely likely that human influence has been the dominant cause of the observed warming since the mid-20th century” (IPCC, 2013). Within the same report the IPCC also stated that, “[g]reenhouse gases contributed a global mean surface warming likely to be in the range of 0.5°C to 1.3°C over the period 1951–2010” (IPCC, 2013). Carbon dioxide emissions, which are a major greenhouse gas and at present are largely due to energy use, have naturally evolved alongside changes in the structure of energy production and consumption. The rigorous study of carbon emissions from energy-use over time, then, is crucial for shaping future carbon dioxide policy domestically. Moreover, insights gained by a case study on the United States’ history of carbon dioxide emissions from energy use (CO₂) are widely applicable to the developing world and emerging market economies.

The present research aids in the understanding of CO₂ emissions at the national level in two unique ways. First, CO₂ emissions are normally studied at their aggregate level instead of the
constituent pollutants that contribute to the overall level of emissions. This study goes beyond the typical analysis of CO\textsubscript{2} emissions as an aggregate and instead focuses at a more ‘micro’ level on one of the largest sources of CO\textsubscript{2} emissions from energy-use for the United States, motor-gasoline. To do this I use a large sample of monthly observations on emissions from motor-gasoline covering the time period 1973-2013. Second, the vast majority of research to date on the nexus of economic growth and CO\textsubscript{2} emissions has assumed that emissions respond the same to changes in income during expansionary periods as contractionary periods. This study relaxes that assumption and explores the possibility of multiple states, or hidden regimes, through the use of a structurally founded Markov regime-switching model. I find that emissions demand is characterized by having two states, and that one state corresponds with periods of aggregate economic turmoil while the other aligns with periods of economic growth.

The relationship among gasoline consumption, gasoline prices, and macroeconomic growth is well studied in the economic literature, though research on these variables and emissions from motor-gasoline is severely limited. Hamilton (1983) is an excellent analysis and survey of the bond between gasoline and economic growth in the post-war period. Rotemberg and Woodford (1996) also comment on the intricate relationship of gasoline prices, demand, and macroeconomic conditions and their effect on markups. Further microeconomic evidence on the relation between aggregate economic shocks and gasoline consumption in Bresnehan and Ramey (1993) indicates that aggregate shocks affect the mix of demand for automobiles. More recently, the dynamic effects of oil-price changes have been analyzed (Lee and Ni, 2002). Specifically, great strides have been made in modelling the oil-macroeconomy relationship as a non-linear relationship (Hamilton 1996, 2003, 2013; Killian and Vigfusson, 2011, 2013). In fact, Hamilton (2003) notes that when
“allocative disturbances are indeed the mechanism whereby oil shocks affect economic activity, then there is no reason to expect a linear relation between oil prices and GDP.” The present study applies these finding by modeling emissions demand from motor-gasoline in a Markov-switching vector autoregression. Using this modeling strategy I am able to capture the asymmetries that have been shown to be important as well as their dynamic effects.

Recognizing that CO$_2$ emissions respond differently to price shocks and changes in income depending on whether or not the economy is in a growth state has important implications for policies aimed at limiting CO$_2$ emissions. Specifically, this finding indicates that permit or tax instruments must be flexible in nature to allow for time-varying demand. In other words, policy instruments must be able to fluctuate in price to reflect the changing nature of demand when the economy is in a downturn so that undue burden is not faced by consumers. This, in essence, is the lesson learned from the Lucas critique.

The rest of the paper continues as follows: section two tests for and discusses the intrinsic deterministic components of CO$_2$ from motor-gasoline, and the relationship of CO$_2$ emissions with other non-stationary variables that will be used in the demand analysis. Section three estimates a non-regime-switching structural supply and demand model of CO$_2$ emissions from motor-gasoline. Section four estimates a structurally founded Markov regime-switching CO$_2$ demand model. Section five concludes with a policy discussion.

2. Data

This section reports the common diagnostics for time-series data that are necessary before a more robust analysis can take place. Included in this analysis are unit-root tests on CO$_2$ from
motor-gasoline, income, and the retail price of gasoline, and a cointegration analysis of these variables.

Data on emissions from motor-gasoline come from the Energy Information Administration (EIA), and are monthly data spanning the time period of 1973:01-2013:06. Data on the average retail price and the wholesale cost, the Free on Board price, also come from the EIA. As a measure of income, real personal income (RPI) is used due to its availability at a monthly frequency. This information comes from the Bureau of Economic Analysis.

Motor-gasoline is perhaps the most familiar source of CO₂ emissions from an individual-use perspective. For that reason, it is quite interesting to note the continued fall in CO₂ emissions from motor-gas since the onset of the 2007 recession. Note, that the EIA’s calculations of CO₂ emissions take account of increased use of ethanol fuel sources and varying energy density and emissions factors which may be a driver of lower pollution levels. What may also be behind continual decreases in motor-gas CO₂ emissions despite the ongoing economic recovery is the increased use of fuel-efficient vehicles by households, firms, and government alike. Because of this continued decline in motor-gas emissions the potential for an environmental Kuznets curve relationship will be examined in the Markov-switching emissions demand analysis to follow.

2.2 Time Series Diagnostics

A battery of tests must first be examined so that the final structural model is accurately specified. First, the Philips-Perron (1988) test for a unit-root is used to measure the deterministic

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2 Hylleberg, et al. (1990) seasonal unit-root tests were evaluated for this data to determine whether or not seasonal variation could be removed. The presence of seasonal unit-roots hold the same deterministic implications as traditional unit-roots, and thusly must be accounted for. An F-test for the presence of seasonal unit-roots is rejected so I choose to use a 12 month moving average to eliminate any excessive “noise” in the data that is simply due to changing conditions in the summer as opposed to winter. Monthly dichotomous variables are included in later regressions to still allow for season specific changes in demand that reach beyond the seasonal average.

3 The literature abounds with EKC related studies. Dinda (2004) is an excellent survey for the unfamiliar reader.
nature of each variable\textsuperscript{4}. The Phillips-Perron test indicates that each series in non-stationary before any transformations have been made, but is stationary after the first difference is taken. Test statistics for the Phillips-Perron test are displayed below in table 1. Following the finding that each series is non-stationary the Johansen (1995, 1998) cointegration test is carried out to explore whether or not the variables can be “cleaned” of the non-stationary process by using the first difference of the series without losing valuable estimation information (Watson, 1994). I find that the variables are not cointegrated, and that the model is reliable when the first difference of the natural log of the variables is used\textsuperscript{5}

\begin{table}[h!]
\centering
\begin{tabular}{lccc}
\hline
Variable & Constant & Constant and Trend & Differenced Series \\
\hline
Motor Gas & -1.168 & -0.305 & -13.905* \\
Income & -0.972 & -0.900 & -25.494* \\
Retail Price & -1.344 & -2.619 & -12.479* \\
\hline
\end{tabular}
\caption{Phillips-Perron Unit-Root Tests}
\end{table}

\textit{Notes:} Test statistics shown. 5 lags are included as per Newey and West (1987). * indicates rejection of the unit-root hypothesis at the 1% level.

3. Structural CO\textsubscript{2} Demand

Because this paper analyzes CO\textsubscript{2} emissions from energy use, the model used here will follow in the vein of previous research on energy demand analysis (Ryan and Ploure, 2009; Burnett, et al., 2013). In order to accurately estimate the demand for emissions from motor-gasoline it is important to separate the effects of price changes due to cost shocks from the effects of price changes due to demand shocks. In other words, one must consider the fact that supply and demand are jointly identified and must account for potential simultaneity in estimation. The

\textsuperscript{4} Augmented Dickey-Fuller tests for a unit-root come to the same conclusion as the Phillips-Perron test.

\textsuperscript{5} The trace statistic is 27.197, which rejects the null hypothesis of cointegration rank above order 0. Minimization of AIC, SBIC, and HQIC indicate that 4 lags should be used in this test.
structural demand model presented here parses out demand-specific and supply-specific shocks by estimating a multi-stage model similar to that of two-stage-least-squares. Specifically, the model is identified by first accounting for asymmetric cost pass-through on the “supply-side” of the system of equations, and then the instrumented price is used in a vector autoregression (VAR) model to account for lagged effects from changes in price or income – the “demand-side.” The structural model is also simultaneously estimated using three-stage-least-squares (3SLS) to prove that the model estimates and standard errors are consistent across estimation strategies. The structural model presented here is intended as a base model from which to compare the Markov regime-switching model to in section four. Note, however, that the supply-side of the model is the same in both the switching and non-switching specifications.

On the supply-side of the model the price of emissions, the retail price of gasoline, is determined by changes in the wholesale cost of gasoline, the Free on Board price. More specifically, the supply-side of the structural model allows for asymmetric cost pass-through via an error-correction model. This type of modelling framework is used because the retail cost of gasoline that consumers face may vary from the wholesale cost of gasoline by differing lagged effects depending on whether the wholesale cost rose or fell in the previous few periods. In essence, the pricing (supply) model allows the price of the pollutant to respond asymmetrically to lagged changes in the wholesale cost of the fossil-fuel. Further, an error correction term is included to capture the cointegrated nature of retail and wholesale prices. This type of modelling framework is very common in the literature on retail gasoline prices (Bachmeier and Griffin, 2003; Lewis and Noel, 2011; Noel and Roach, 2014). The supply side of the system is shown below in equation (1)

\[
\Delta p_t = \gamma_0 + \sum_{j=0}^{J-1} (\alpha_j^+ \Delta C_{t-j}^+ + \alpha_j^- \Delta C_{t-j}^-) + \sum_{j=1}^{J-1} (\beta_j^+ \Delta p_{t-j}^+ + \beta_j^- \Delta p_{t-j}^-) + \theta z_t + \epsilon_t
\]
With,

\[ z_t = p_{t-1} - C_{t-1} \]
\[ \Delta C_t = C_t - C_{t-1} \]
\[ \Delta p_t = p_t - p_{t-1} \]

Where \( C_t \) denotes the wholesale cost of gasoline at time \( t \); variables with a plus (minus) sign indicate an increase (decrease) from the previous period, explained below in equation (2); and \( z_t \), the error correction term, is the residual of an OLS regression of prices on costs in the previous period.

\[ \Delta C_t^+ = \max[0, \Delta C_t] \]
\[ \Delta C_t^- = \min[0, \Delta C_t] \]
\[ \Delta p_t^+ = \max[0, \Delta p_t] \]
\[ \Delta p_t^- = \min[0, \Delta p_t] \]

Following the supply-side estimation, the instrumented retail price of gasoline is used in the demand-side of the system. In order to model CO\(_2\) demand a vector autoregression (VAR) model is estimated so that the intricacies of the serially correlated variables may be accounted for. The VAR system is displayed below in equation (3)

\[ Y_t = a + \sum_{i=1}^{P} B_i Y_{t-i} + \sum_{j=1}^{M} \beta_j Season_j + \epsilon_t \]
where $Y_t$ is a matrix containing the log-difference of CO$_2$ emissions from motor-gasoline ($\Delta CO_2_t$), the log-difference of instrumented price from the supply side of the system ($\Delta p_t$), and the log-difference of real personal income ($\Delta M_t$). Though the generic form of the VAR is shown in equation (3) minimization of information criteria implies that the optimal lag length of the VAR is three periods. Estimation results are presented below in table 2. Note that seasonal effects are captured with the inclusion of monthly dummy variables in both estimation methods, though not displayed alongside the rest of the estimated parameters. Only the CO$_2$ demand estimates are shown for the VAR specification.

As a robustness check the system is also simultaneously estimated using the method of three-stage-least-squares (3SLS). It is possible that there is correlation among the error terms across equations and failure to account for such behavior would lead to inconsistent or inefficient estimates. The method of 3SLS can improve upon the efficiency of the estimated results by estimating a separate covariance matrix that accounts for correlation in the disturbances. This is the additional stage in the estimation process beyond the regression by regression technique that is common to two-stage-least-squares, as the VAR model with instrumented price may be interpreted as. I find that the point estimates and standard errors are extremely consistent across specifications and that using a VAR model for demand as the “second stage” yields reliable results. This finding motivates the two-stage method below when the full Markov-switching model is estimated.

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6 The full set of parameter estimates is available from the author on request.
### TABLE 2 – STRUCTURAL MODEL ESTIMATES

<table>
<thead>
<tr>
<th>Demand</th>
<th>Dependent variable: $\Delta CO_2_t$</th>
<th>VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3SLS</td>
<td></td>
</tr>
<tr>
<td>$\Delta CO_2_{t-1}$</td>
<td>0.1335***</td>
<td>(0.0443)</td>
</tr>
<tr>
<td>$\Delta CO_2_{t-2}$</td>
<td>0.2894***</td>
<td>(0.0426)</td>
</tr>
<tr>
<td>$\Delta CO_2_{t-3}$</td>
<td>0.3448***</td>
<td>(0.0440)</td>
</tr>
<tr>
<td>$\Delta p_{t-1}$</td>
<td>-0.0042*</td>
<td>(0.0024)</td>
</tr>
<tr>
<td>$\Delta p_{t-2}$</td>
<td>0.0024</td>
<td>(0.0026)</td>
</tr>
<tr>
<td>$\Delta p_{t-3}$</td>
<td>-0.0039*</td>
<td>(0.0023)</td>
</tr>
<tr>
<td>$\Delta M_{t-1}$</td>
<td>0.0443**</td>
<td>(0.0175)</td>
</tr>
<tr>
<td>$\Delta M_{t-2}$</td>
<td>0.0065</td>
<td>(0.0180)</td>
</tr>
<tr>
<td>$\Delta M_{t-3}$</td>
<td>-0.0034</td>
<td>(0.0177)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supply</th>
<th>Dependent variable: $\Delta p_t$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta CO_2_t$</td>
<td>0.8410</td>
<td>(1.1762)</td>
</tr>
<tr>
<td>$\Delta p^{+}_{t-1}$</td>
<td>0.2627***</td>
<td>(0.0699)</td>
</tr>
<tr>
<td>$\Delta p^{-}_{t-2}$</td>
<td>-0.1248**</td>
<td>(0.0634)</td>
</tr>
<tr>
<td>$\Delta p^{-}_{t-1}$</td>
<td>0.4562***</td>
<td>(0.0764)</td>
</tr>
<tr>
<td>$\Delta p^{-}_{t-2}$</td>
<td>-0.2684***</td>
<td>(0.0655)</td>
</tr>
<tr>
<td>$\Delta C^+_{t}$</td>
<td>0.2427***</td>
<td>(0.0438)</td>
</tr>
<tr>
<td>$\Delta C^+_{t-1}$</td>
<td>0.2214***</td>
<td>(0.0491)</td>
</tr>
<tr>
<td>$\Delta C^+_{t-2}$</td>
<td>-0.0223</td>
<td>(0.0505)</td>
</tr>
<tr>
<td>$\Delta C^-_{t}$</td>
<td>0.2544***</td>
<td>(0.0405)</td>
</tr>
<tr>
<td>$\Delta C^-_{t-1}$</td>
<td>0.1543***</td>
<td>(0.0503)</td>
</tr>
<tr>
<td>$\Delta C^-_{t-2}$</td>
<td>-0.0329</td>
<td>(0.0527)</td>
</tr>
<tr>
<td>$p_{t-1}$</td>
<td>-0.0319***</td>
<td>(0.0115)</td>
</tr>
<tr>
<td>$C_{t-1}$</td>
<td>0.0228***</td>
<td>(0.0088)</td>
</tr>
</tbody>
</table>

Demand R² 0.4411 Demand R² 0.4587
Supply R² 0.5824 Supply R² 0.5848

Notes: Statistical significance at the 1%, 5%, and 10% level denoted by ***, **, *, respectively.

Analysis of the marginal effects for each variable is difficult and yields little in terms of relevant policy prescriptions due to non-linearity in the supply-side of the system, and the fact that each of the variables is in lagged log-differences on the demand side. On the demand side, the estimated coefficients show the marginal effect of an increase in the growth rate of income or the
growth rate of the price of gasoline. In order to comment on the effect of a change in either income or the price of gasoline the cumulative response to a shock in either variable is calculated and displayed in figures 1 and 2. The impulse response functions displayed below are given structure (orthogonalized) assuming the following Choleski decomposition of the error term.

\[
\varepsilon_t \equiv \begin{pmatrix}
\varepsilon_t^M \\
\varepsilon_t^P \\
\varepsilon_t^{CO_2}
\end{pmatrix} = \begin{bmatrix}
a_{11} & 0 & 0 \\
a_{21} & a_{22} & 0 \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_t^{\text{real income shock}} \\
\varepsilon_t^{\text{gasoline price shock}} \\
\varepsilon_t^{\text{CO}_2 \text{ demand shock}}
\end{pmatrix}
\]

Here it is assumed that income shocks can affect CO\(_2\) demand and the price of gasoline contemporaneously, but the converse is not true. Similarly a shock to the retail price of gasoline affects CO\(_2\) in the current period, but CO\(_2\) demand does not have a contemporaneous effect. The interpretation of the impulse responses displayed below does not change qualitatively under a different structural ordering.
Figure 2 – CO$_2$ Response to a Shock in Price

- 95% Confidence
- Cumulative Orthogonalized IRF

Months:
- 0
- 3
- 6
- 9
- 12
From the orthogonalized impulse responses I find that a positive shock to income causes an increase in emissions from motor-gasoline, though this can only be distinguished from zero for the first few periods. Similarly, analysis of the cumulative impulse response function for a shock to the energy price shows that a positive shock to the price of the energy will cause a decrease in the amount of emissions, albeit by a very small amount. Both of these results are as expected, and align with similar findings in the literature on price and income shocks on gasoline demand.

4. Markov Switching CO$_2$ Demand

If we are to follow the Lucas critique, then we cannot assume that the marginal effects of price or income changes are constant across different policy regimes$^7$. To that end, this section explores whether the effects of economic growth and energy price swings have changed over time for CO$_2$ emissions from motor-gasoline. The primary hypothesis explored here is whether or not the marginal effect of changes in the retail price of gasoline or changes in the economic growth variable, real personal income, are state dependent$^8$.

Although the practice of analyzing a time-series in a Markov regime switching framework has mostly been used for financial data$^9$ the approach is applicable to CO$_2$ emissions from energy at long horizons because macroeconomic fluctuations can certainly effect emissions depending on whether the economy is in an expansionary or contracting state. Thus, a Markov regime-switching model is estimated to allow for the potential of state-dependent coefficients.

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$^7$ Robbins (1932) offers a (much) earlier discussion on why it is important to account for structural change.

$^8$ I use the terminology “regime” and “state” interchangeably.

$^9$ e.g. comparing price fluctuations in bull and bear markets.
Markov switching (MS) models are certainly not new to the field of econometrics, and have been used since the primal work of Goldfeld and Quant (1973), though they became widely popular after the work of Hamilton (1989). Since then, though, there have been relatively few applications that study gasoline demand. Moreover, previous studies have not focused on pollution and instead have primarily been concerned with the effect of energy price shocks on macroeconomic aggregates (Cologni and Manera, 2009; Hamilton, 1996, 2003) or gasoline price dynamics in the industrial organization literature (Noel, 2007, 2009, 2012). There are a few studies with which the present analysis results can be compared to. Soytas, et al. (2007) study the relationship between emissions from energy and income in the United States while paying particular attention to the time-series properties of emissions. Roach (2013) proxies for vehicle emissions by estimating the impact of income shocks on the amount of miles driven by vehicles. Galeotti, et al. (2006) offers a “robustness exercise” in that they use a non-parametric functional form to study the income-CO₂ relationship. Doda (2014) finds that the cyclical component of emissions is positively correlated with the cyclical component of GDP. The only other work that uses a MS framework to describe CO₂ emissions and economic growth is Park and Hong’s (2013) analysis of South Korea. To the author’s knowledge this study is the first to consider CO₂ emissions in a MS framework for the United States. In a somewhat related study, Chevallier (2011a, 2011b) studies the futures price for the European Union Allowance, a carbon price of sorts, in a two-state Markov switching model. The present study may be seen as a complement to Chevallier’s work on carbon prices for two reasons. First, carbon allowance prices are naturally linked to the quantity of carbon demanded in a society. Second, because there is not a Federal-level policy for limiting CO₂ emissions in the United States this study allows for a better
understanding of the underlying determinants should a federal tax or permit system be implemented like that of the European Union. Further, this paper also employs a two-state Markov regime switching model to capture the varying demand in high-growth and low-growth regimes.

While a latent regime analysis is useful in analyzing the potential for asymmetric marginal affects, it is also useful to explore a more divisive topic – whether or not a structural change has transformed how people view the environment after reaching some threshold level of income. This type of psychological shift in demand is obviously hidden from the observer, but is often proxied for by allowing income to enter the regression equation non-linearly\(^\text{10}\). A common retort to this hypothesis is that perhaps environmental quality and income are not causally related but simply correlated over time (Carson, 2010). The hidden regime analysis discussed below allows for this question to be analyzed in a novel way that does not depend on transformations of the income variable. This is achieved by estimating the model with more than two regime variables.

4.1 Univariate Markov-Switching Analysis

A first look at the underlying regime possibilities is examined here. A more detailed description of how the model is solved is provided below when the full model is estimated. For now, this model will serve to set a benchmark for the emissions time-series. I find that for the emissions series without covariates there are only two states that are plausible. These states very closely align with changes in aggregate economic activity and do not indicate that a structural change for CO\(_2\) from motor-gasoline has occurred. This finding drives the main specification of

\(^{10}\) This is known as the environmental Kuznets curve (EKC), and has been studied extensively in the environmental economics literature. The most highly cited example of this modeling framework is Grossman and Krueger (1991), though many other studies have explored the EKC in this way.
two latent regimes, expansionary and contractionary, explored below for the structural MS-VAR model. The univariate MS model is expressed as,

\[
\begin{align*}
y_t &= \mu_{S_t} + \epsilon_t \\
\epsilon_t &\sim N(0, \sigma_{S_t}^2) \\
S_t &= 1, \ldots, k
\end{align*}
\]

where \( y_t \) is the growth rate of CO\(_2\) emissions from motor-gasoline\(^{11}\), and both the mean and variance of the series are state dependent.

The probability of switching from one regime to another is an important element of the MS framework. Because the transition between states is a stochastic process, a transition matrix that includes information on the probability of a regime switch is estimated and takes the form,

\[
P = \begin{pmatrix}
  p_{i,i} & \cdots & p_{i,k} \\
  \vdots & \ddots & \vdots \\
  p_{k,i} & \cdots & p_{k,k}
\end{pmatrix}
\]

where a transition from state \( k \) to state \( i \) is represented by the value of \( p_{i,k} \), and the sum of each column is equal to one to satisfy the adding-up constraint. For the base specification, the CO\(_2\) time-series without covariates, the proper amount of regimes as indicated by statistical significance of the switching parameters is found to be two.

The smoothed transition probabilities are graphed below in figure 3 with NBER recession dates shown with gray bars. Recession dates are shown to illuminate the relationship between regime changes and recessionary times. Although this will be explored in much more rigor in the

\(^{11}\) The log-difference of CO\(_2\) from motor gasoline.
following section, anecdotal evidence shows that emissions are very responsive to macroeconomic conditions.

From the smoothed transition probabilities displayed in figure 3 it appears that the regimes can be clearly be demarked as expansionary and contractionary periods. In general, emissions from motor-gasoline are relatively calm and do not switch regimes over the period 1983-2007 with the exception of a regime switch for the recession beginning in the third quarter of 1990. This finding makes intuitive sense as this is a time period known for moderation. An interesting finding can be seen in the smoothed probabilities following the most recent recession ending in the second quarter of 2009. During this time period the smoothed transition probabilities give the impression that the
United States is still in a recessionary state. This could perhaps be due to the relatively low amount of economic growth occurring during this time, though this finding could also indicate that a new state of the economy exists in which economic growth proceeds, but emissions demand wanes. This hypothesis is explored and refuted, though, in the following section.

4.2 *Multivariate Markov-Switching Analysis*

Following the structural model presented in section three, a Markov regime-switching vector auto-regression (MS-VAR) model is estimated to fully characterize the dynamic nature of CO₂ demand. Specifically, this model allows for state-dependent correlation among the variables over time. The general form of the MS-VAR model is displayed below in equation (7).

\[ Y_t = B_{0,S_t} + \sum_{i=1}^{3} B_{S_t} Y_{t-i} + \epsilon_t \]

where the subscript \( S_t \) denotes the coefficient value in state \( k \in \{1, ..., K\} \), each variable in the system, \( Y_t \), is in log-differences, and the price variable is the estimated price from the first stage error-correction model\(^{12}\). The regime-switching error term for the model is represented in equation (8), below.

\[ \epsilon_t \sim N(0, \Sigma_{S_t}) \]

\[ \Sigma_{S_t} = \begin{pmatrix} \sigma_{1,S_t}^2 & \cdots & \sigma_{1,3,S_t} \\ \vdots & \ddots & \vdots \\ \sigma_{1,3,S_t} & \cdots & \sigma_{3,S_t}^2 \end{pmatrix} \]

\(^{12}\) A total of three lags is used following the standard practice of minimization of information criteria. Further, likelihood ratio tests indicate that this model is preferred to specifications with fewer explanatory variables. Note that with each included lag an additional 18 parameters must be estimated. The final model has 68 estimated parameters.
Though not shown explicitly in the tables below, the model supports state-dependent error terms at the 1% significance level. The model is solved by iterative maximization of the log-likelihood function, (9) shown below, following Hamilton (1989, 1994) and Perlin (2010).

\[
\ln \mathcal{L} = \sum_{t=1}^{T} \ln \sum_{j=1}^{K} (f(CO2_{it} | S_t = j, \theta) \Pr(S_t = j | \psi_t))
\]

where \(S_t\) is an ergodic Markov chain that is aperiodic and reoccurs throughout the sample. Note that the probability of switching states is constrained to be between 0 and 1, and designed such that the probability of switching from state \(k\) to any other state, including \(k\), adds up to 1. Further, it bears mentioning that the smoothed transition probabilities that are estimated are not the probability that an observation is in state \(k\), \textit{per se}, but instead the probability that the series will transition to state \(k\) from the current state. As an example, the smoothed transition probabilities show the probability of transitioning to a growth state in the following period rather than the probability that an observation is currently in a growth state.

Figure 4, below, shows the smoothed transition probabilities for the two-state MS-VAR model with NBER recessions denoted as before. As in the univariate analysis, the two states found in the full model very closely align with changes in aggregate economic activity. The demand for CO\(_2\) emissions from motor-gasoline responds asymmetrically to changes in the retail price of gasoline or income depending on whether or not the economy is in an expansionary or contracting state. This is an important finding because it certainly impacts the way climate policy ought to be formed. It is interesting to note that the multivariate model is able to distinguish between the 2007
recession period and the period of growth that followed while the previous univariate model could not.

As a host of the aforementioned authors have shown, oil-price shocks contribute greatly to aggregate economic instability. Indeed, a MS-VAR model is used instead of a simple one-period model so that asymmetric lagged effects may contribute to changes in real personal income as well as changes in emissions demand. Further, it is plainly obvious that real personal income and aggregate economic activity are linked. It may be illuminating, then, to compare both the income and retail price series against the probability of transitioning to the contracting state in order to see the correlation between shocks to energy or income and the existence of the contracting state.
Further, this is a first look at whether or not the latent regimes found in this analysis are truly hidden to the econometrician, and if they are necessary in explaining CO₂ emissions at all. Figures 5 and 6, below, display each series alongside the smoothed transition probability of the state that aligns with periods of contraction.

**Figure 5 – Income and Smoothed Contracting State Probability**

![Graph showing income and contracting state probabilities over time.](image-url)
Each figure shows that income or price shocks are aligned with the probability of transitioning to the contracting state, but neither series fully explains the existence of the contracting state. For instance, there are periods in which there is a large shock to income and the probability of transitioning to a contracting state increases, and there are times in which no such movement occurs. The same can be said about price shocks. Although this is anecdotal evidence on the existence of multiple regimes, the robustness analysis that follows confirms that these two regimes accurately portray emissions demand.

4.3 Regime Classification Robustness

This section provides an analysis of the robustness of the two-regime model. Specifically, this section addresses two contrasting hypotheses: first, whether or not further states can be tolerated in the regime switching model; and second, whether or not a two-regime model is even
appropriate for the data at hand. For the first hypothesis an analysis of the fit of the model and the evidence of significant third and fourth state variables will be used. For the second hypothesis the Ang and Bekaert (2002) definition for regime classification is considered.

Statistics for both the two-state and three-state MS-VAR models are presented in table 2, below. Included in this table are various diagnostic statistics as well as the estimated transition matrix and the expected duration of each regime. Parameter estimates for the full model are not shown because a total of 68 different parameters are estimated for the two-state model, and 105 for the three-state model\textsuperscript{13}.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Two-State</th>
<th>Three-State</th>
</tr>
</thead>
<tbody>
<tr>
<td>R\textsuperscript{2}</td>
<td>0.5379</td>
<td>0.5644</td>
</tr>
<tr>
<td>Adjusted R\textsuperscript{2}</td>
<td>0.4332</td>
<td>0.3918</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>4982.81</td>
<td>5046.91</td>
</tr>
<tr>
<td>RCM</td>
<td>7.3023</td>
<td>0.4309</td>
</tr>
<tr>
<td>Expected duration S\textsubscript{1}</td>
<td>4.29</td>
<td>6.09</td>
</tr>
<tr>
<td>Expected duration S\textsubscript{2}</td>
<td>31.50</td>
<td>1.98</td>
</tr>
<tr>
<td>Expected duration S\textsubscript{3}</td>
<td>-</td>
<td>3.87</td>
</tr>
<tr>
<td>Transition Matrix</td>
<td>(\begin{pmatrix} 0.77 &amp; 0.03 \ 0.23 &amp; 0.97 \end{pmatrix})</td>
<td>(\begin{pmatrix} 0.84 &amp; 0.51 &amp; 0.26 \ 0.12 &amp; 0.49 &amp; 0.00 \ 0.04 &amp; 0.00 &amp; 0.74 \end{pmatrix})</td>
</tr>
</tbody>
</table>

Considering Hamilton’s original (1989) application that brought about wide-spread appeal for this technique there is an intuitive appeal to include only two regimes in the MS model to track the boom and bust tendencies of emissions. Regardless, the possibility that additional regimes exist

\textsuperscript{13} Parameter estimates for each model are available upon request.
should still be examined. Moreover, the inclusion of additional regimes could allow for a type of structural shift in emissions demand that could not be observed by the researcher; for instance lower emissions demand after a threshold level of income as the environmental Kuznets curve hypothesis posits. The structural model presented here does not support the presence of more than two states. The significance of individual switching parameters is first considered in the rejection of the three-state model. Of the 105 estimated parameters in the three-state model, none of the third-state parameters were found to be statistically significant. Further, the smoothed regime-switching probability of the second state is effectively zero for the entire sample period. As with the three-regime model the four-regime model yields insignificant model parameters and zero probability of switching to either the third and fourth regimes. Thus, the four-regime model is also dismissed as a possibility.

Evidence against the three-state model can also be seen by the fit of the model with two states compared to the fit of the model with three states. The fit statistic used here is the coefficient of determination proposed in Krolzig (1997) that accounts for the bias towards accepting models with more parameters, an adjusted $R^2$ of sorts. The $R^2$ for the three-state CO$_2$ demand model is 0.56 and the adjusted $R^2$ is 0.39. The fact that the adjusted $R^2$ is lower is unsurprising given the transition probabilities and lack of significance of the third-state coefficients. The $R^2$ for the two-state model is slightly less than the three-state model without adjustment, but the adjusted $R^2$ is larger than the three-state model at 0.43. Thus, after accounting for differences in the degrees of freedom the two-state model does a better job of explaining the variation in CO$_2$ demand.

In order to determine how well the two-regime model defines demand for CO$_2$ emissions from motor-gasoline the Ang and Bekaert (2002) measure of regime classifications is utilized.
Note, that standard likelihood ratio (LR) based tests of nested models is not applicable in MS-VAR models (Krolzig, 1997; Garcia 1998). This is due to the fact that when the number of regimes is unknown, the asymptotic distribution of the LR test is non-standard. The Ang and Bekaert method involves testing whether or not the model is able to categorize regimes “sharply” using a Regime Classification Measure (RCM). The RCM is formulated as follows for $K$ states,

\[
\text{RCM}(K) = 100K^2 \frac{1}{T} \sum_{t=1}^{T} (\prod_{i=1}^{K} p_{it})
\]

Where $p_{it}$ is the smoothed transition probability, and the terms to the left of the summation operator normalize the RCM statistic to be between 0 and 100. Ang and Bekart note that an ideal regime classification would yield a statistic of 0, and that if there is no information on the regimes revealed through the model estimates the statistic would be equal to 100. Common practice uses a value of 50 as the benchmark for whether or not the model is accurately explained by the presence of $k$ states (Chan, et al. 2011; Chevallier, 2011). The RCM measure for the two-state motor gasoline demand model is quite low with a value of 7.3023. This signifies that the model characterizes the hidden regime switches very sharply\textsuperscript{14}.

5. Conclusions, Policy Discussion

\textsuperscript{14} The RCM measure for the model with three states is very low at 0.4. This is not due to the sharpness of the regime classifications, but rather by the third state having a smoothed probability of 0 for the vast majority of the sample period, and the sharp definition of the other two-states.
Given the recent findings of the IPCC that greenhouse gases from anthropogenic sources are “very likely” leading to increased global warming, a more thorough understanding of the properties and determinants of CO₂ emissions from one of the world’s leading emitters is very important. To that end, this paper has furthered the research on CO₂ emissions in general, and on emissions in the United States in particular, in two distinct ways.

First, focusing on a distinct energy source that contributes to the overall level of aggregate emissions allows for more nuanced and specific policy proposals to be developed. Moreover, analyzing a specific polluting source brings to light issues that may not be apparent when exploring the determinants of emissions in aggregate. For instance, the structural model presented here has highlighted the importance that contemporaneous and past prices of both retail gasoline and the wholesale price of gasoline have in shaping the quantity demanded of gasoline, and hence the amount of CO₂ emissions from gasoline.

Second, this paper has exhibited the importance of allowing for regime-dependent coefficients when explaining the demand for CO₂ emissions at the national level. This finding has vital policy implications when considering policy instruments meant to reduce the amount of emissions from motor-gasoline. Following the lessons of the Lucas critique, the existence of hidden regime shifts implies that policy instruments meant to deter CO₂ emissions ought to be flexible in nature in order to minimize any unintended burden from the policy. Further, the use of flexible policy instruments will lessen unintended consequences that may arise following the adoption of a CO₂ policy. Bankable and transferable permits that are sold in competitive markets are, by design, responsive to hidden demand parameters that may be unobservable to the policy-maker through the price mechanism. For instance, one would expect that the demand, and hence
the price, of transferable permits to fall during a recession because firms require less permits while using less energy. Given the findings in this paper, bankable and transferable permits seem to be a promising avenue for CO\textsubscript{2} mitigation. Concerning price-based instruments, the findings of this paper support a dynamic tax that fluctuates in accordance with aggregate economic activity.

6. References


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